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Laser Forming of Varying Thickness Plate—Part I: Process Analysis

High-intensity laser beams can be used to heat and bend metal plates, but the mechanisms of the laser forming (LF) process are not well understood or precisely controllable. The objective of the National Institute of Standards and Technology sponsored project "Laser Forming of Complex Structures" is to develop technologies for a controllable, repeatable laser forming process that shapes and reshapes a wide range of complex structures such as compressor airfoils that are complex 3D geometries with large thickness variation. In order to apply laser forming to complex 3D geometries, the process analysis and process synthesis (design process parameters such as scanning paths and heating conditions for a desired shape) of LF of varying thickness plate are conducted in this paper. In this study, experimental, numerical, and analytical methods are used to investigate the bending mechanism and parametric effects on the deformation characteristics of varying thickness plates. A transition of the laser forming mechanism was found to occur along the scanning path when the thickness varies. The effect of scanning speed, beam spot size, and multiple scanning on the degree of bending was investigated. The proposed analytical model can predict the bending angle and angle variations for laser forming of varying thickness plate.
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Keywords: laser forming, varying-thickness, bending mechanism, analytical model

1 Introduction

Laser forming (LF) is a nontraditional forming process that does not require hard tooling or external forces and hence, may dramatically increase process flexibility and reduce the cost of forming process. It promises to become an inexpensive and flexible mainstream manufacturing process both for rapid prototyping and manufacturing applications. A typical application might call for small secondary deformations for shape tuning or distortion correction of a formed, machined, or welded component [1]. Compressor airfoil shape tuning by LF is one of the objectives in the National Institute of Standards and Technology (NIST)-sponsored project "Laser Forming of Complex Structures." To implement LF in the shape tuning of compressor airfoils, complex 3D geometries with large thickness and curvature variation, the analysis and process design of LF on varying thickness plate was initiated with the aim to improve the understanding of the basic underlying deformation mechanisms.

Numerous investigations of LF processes have been carried out to better understand process mechanisms and effects of key process parameters on the deformations and mechanical properties of formed parts [2,3]. Geometric effects, especially due to thickness, play an important role in laser forming. For example, Scully [4] proposed that plate thickness is one of the primary factors in laser forming (two other primary factors being laser power and scanning speed), and Vollertsen [5,6] proposed a simple two-layer model based on a temperature gradient mechanism (TGM), in which he found the bending deformation to be inversely proportional to the square of the plate thickness.

Cheng et al. [7] also proposed an analytical model to predict bending variation due to the change of plate size (width and length). However, in all the above analysis, a uniform thickness was considered, and the theories cannot be applied to varying thickness plates.

Nevertheless, attempts have been made to analyze varying thickness plates, but analytical solutions have been sparse and ad hoc due to inherent difficulties in mathematical treatment [8]. For example, Conway [9] studied the bending of circular plates with radial thickness variation. Petrina and Conway [10] reported data on a rectangular plate problem with thickness varying exponentially in one direction. Zenkour [11] gave an exact solution for the bending of thin rectangular plates with uniform, linear, and quadratic thickness variation based on classical thin-plate theory. But for the bending of varying thickness plate caused by laser irradiation, due to the complicated thermoelastoplastic process mechanism, few references have been found.

In this paper, experiments and simulations of LF on thin plate with varying thickness were carried out. The characteristics of bending deformation subject to centerline and diagonal scans were investigated in detail. The evolution of bending variation under various scanning speeds and beam spot sizes is also investigated. To predict bending angle, angle variation, and to characterize the effect of heating conditions on varying thickness plate, a further analysis is conducted which accounts for the moving heat source and changing bending rigidity. The obtained understanding will be applied in process design of laser forming of typical 3D shapes with varying thickness in Part II of this paper.

2 Experiment and Simulation Conditions

The experiment and simulation were carried out on plates with linear thickness variation from 1 mm edge to 3 mm center thickness as shown in Fig. 1. The simplified shape was chosen because the authors want to clarify the effect of laser forming processes on varying thickness without inducing other complexities and at the same time without loss of generality. The material is low carbon steel AISI-1010. The tapered-thickness shape is machined by a milling operation, and then annealed to release the residual

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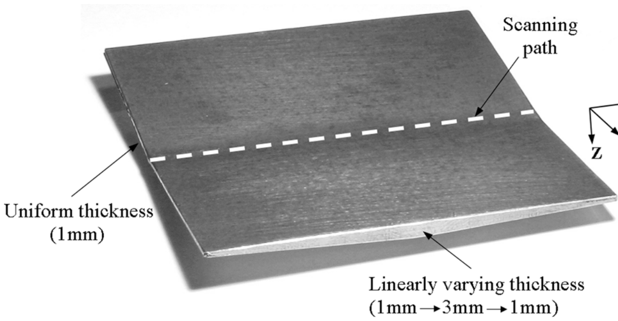

Fig. 1 Laser formed varying thickness plate showing coordinate system (scanned along the centerline, plate size: 80 ×80 mm, power=1000 W, scanning speed=20 mm/s, beam spot size=8 mm, number of scans=10)

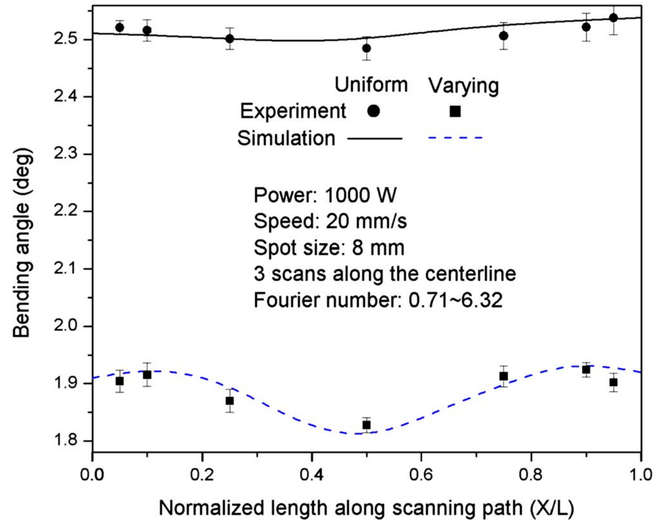

Fig. 2 Bending angle variation of varying thickness plate and uniform-thickness (*h*=2 mm) plate along the scanning direction

stresses. The straight-line laser forming, in which the scanning path is either along the centerline or along the diagonal line, was conducted with a PRC-1500 CO_2 laser system, which has a maximum output power of 1500 W. The laser operates in continuous wave mode and the power density distribution of the laser beam follows a Gaussian function (TEM_{00}). The irradiated surface is coated with graphite to enhance the absorption of incident laser energy. For multiple scans, graphite was recoated after every single scan to obtain the same heat input at every single scan, and the scans were conducted along the same scanning path with alternate directions in order to reduce the edge effect.

Under constant heating conditions, plate thickness plays an important role in determining the laser forming mechanism, and scanning along a varying thickness plate may result in a transition in bending mechanisms. The temperature gradient mechanism (TGM), the most widely reported laser-bending mechanism, is where rapid surface heating by a laser beam and slow heat conduction into the plate produces a steep thermal gradient into the material, which results in differential thermal expansion and plastic deformation through the thickness, generating a net bending toward the heat source. In the buckling mechanism (BM), there is no steep temperature gradient through the plate thickness and bending can be either toward or away from the laser beam depending on a number of factors. In the upsetting mechanism (UM), the plate is compressed with an almost constant strain through the thickness, causing localized shortening and an increase in thickness.

Some of above mechanisms can accompany each other or switch from one mechanism to another based on the local conditions. The TGM is dominant under conditions corresponding to a smaller modified Fourier number $F_0=\alpha \cdot d/(h^2 \cdot v)$, where α is thermal diffusivity, d the beam diameter at the workpiece surface, h the plate thickness, and v the scanning speed [2]. The BM or UM dominates for a high Fourier number. Since the thickness plays prominently in calculation of the Fourier number, it can be reasoned that there may be a transition between TGM and BM or UM on a varying thickness plate even as the heating parameters are constant along the scanning path.

In the present paper, the LF simulation was carried out in ABAQUS where material properties such as the modulus of elasticity, heat transfer properties, thermal conductivity, specific heat, and flow stress were temperature dependent. A strain-hardening coefficient, also temperature dependent, was defined in order to consider strain hardening of the material. Further, no melting is involved in the forming process, and the 20-node element C3D20 was used due to its resistance to shear locking and hourglass effects, making it suitable for a bending-deformation-dominated process such as laser forming.

3 Results and Discussion

3.1 Scanning Along the Centerline. Figure 2 shows the bending angle of a varying thickness plate and an equivalent uniform thickness (h=2 mm) plate under the LF condition of P =1000 W, V=20 mm/s, and spot size=8 mm. Compared with uniform-thickness plate, the variation of bending angle in the varying thickness plate is more obvious. The nonuniformity of deformation is not only due to the variation of bending rigidity, but also due to the variation of the temperature field generated in the plate. Figure 3 shows comparison of peak temperature distribution between varying thickness plate and uniform thickness plate. For both cases, the peak temperature is lower at the entrance and higher at the exit of the scanning path due to the edge effect [12]. For uniform thickness plate, the peak temperature is nearly constant along the scanning path except at the two ends. For varying thickness plate, the peak temperature on both the scanned and opposite (unscanned) surface drops with increasing thickness due to an increasing heat sink effect. It is noted that the peak tempera-

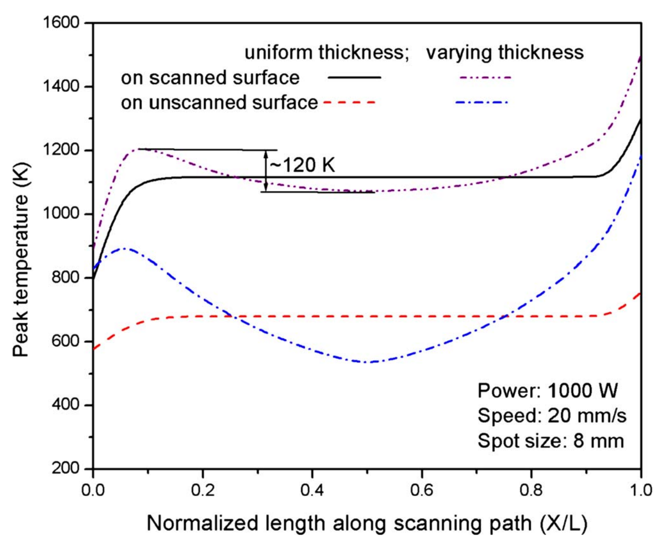

Fig. 3 Comparison of the temperature distribution along the scanning path between varying thickness plate and equivalent-uniform thickness (*h*=2 mm) plate

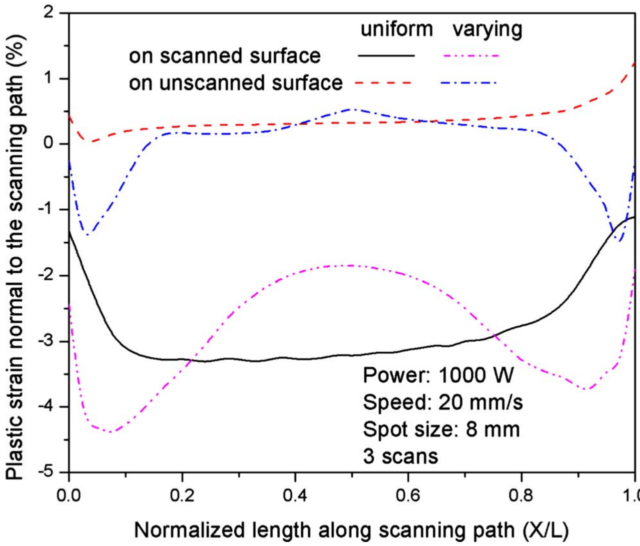

Fig. 4 Comparison of the plastic strain normal to the heating direction between varying thickness plate and equivalent-uniform thickness (*h*=2 mm) plate

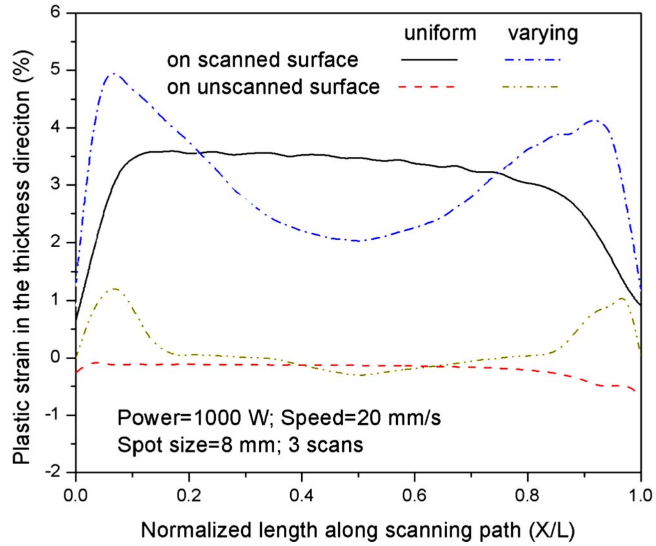

Fig. 5 The distribution of plastic strain in thickness direction along the scanning path

ture drop on the unscanned surface is much larger than that of the scanned surface, and thus the temperature gradient through the thickness direction increases with the increasing plate thickness (see Fig. 3).

From the bending variation of the varying thickness specimen shown in Fig. 2, it is also seen that the bending angle increases first then decreases from the two ends to the center of the scanning line. The reason can be stated as follows. In general, bending deformation is determined by temperature gradient, peak temperature, and bending rigidity. When thickness increases from the end to the center of the plate, the peak temperature decreases while the temperature gradient and bending rigidity increase. The increase of temperature gradient is helpful to achieve large deformation, however, the reduced peak temperature and the increase of bending rigidity will reduce the bending deformation. The interaction between these three effects finally causes the above bending angle trend. From the distribution of plastic strain in the direction perpendicular to the scanning path (shown in Fig. 4), it is clearly seen that the bending deformation increases first then decreases from the ends to the center, which agrees well with the bending angle trend measured by experiment.

If the modified Fourier number is used to approximately estimate the laser forming mechanism under the above conditions ($\alpha=15.75$ mm/s, $d=8$ mm, $v=20$ mm/s), it will be found that F_0 varies from 6.3 to 0.7 when the thickness varies from 1 mm to 3 mm indicating a possible transition of mechanism along the scanning line. From the temperature distribution, it is already found that there is no steep temperature gradient through the thickness at thinner locations; in addition, from the distribution of plastic strain in the through-thickness direction (Fig. 5), it is shown larger thickness increases occur at the thinner locations. Therefore, it can be concluded that UM dominates at the thinner ends and TGM dominates at the thicker center. For constant thickness plate, the bending mechanism will not change if the laser power, scanning speed and beam spot size are constant.

The plastic strains in the middle surface are defined as in-plane strains that expand or contract the middle surface. The difference of plastic strain between the scanned surface and the middle surface is bending strain that bends the plate toward the scanned surface, as shown in Fig. 6. Figure 7 shows the distribution of in-plane strain and the ratio of bending strain to in-plane strain between varying thickness plate and equivalent uniform-thickness ($h=2$ mm) plate. It is seen that the ratio value increases with the thickness, which means the effect of bending strain plays more a important role in bending the thicker positions of the plate. This is consistent with the previous analysis that TGM is dominant in the thicker positions. More detail about the characteristics of in-plane and bending strain will be discussed in Part II of this paper.

3.2 Scanning Along the Diagonal Line. Figure 8 shows the resulting geometry of a varying thickness sample scanned along the diagonal line. As in the previous example, under this scanning scheme the heat dissipation condition varies as well as the bending rigidity along the scanning path. The measured and simulated bending angle variations are shown in Fig. 9 for both uniform thickness ($h=2$ mm) and varying thickness diagonally scanned plates. It is seen that the bending angle of the uniform thickness plate is larger than that of the varying thickness plate, and the bending variation of the varying thickness plate is much larger than that of the uniform thick plate. The reason is the same with that of the plate scanned along the centerline that is discussed in

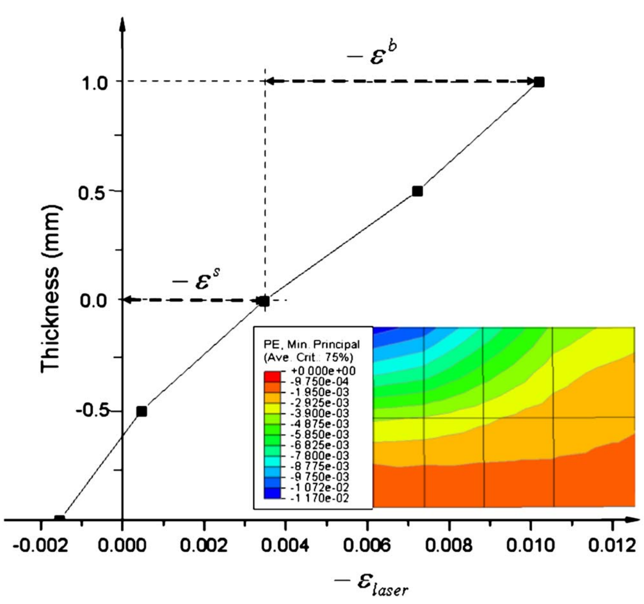

Fig. 6 Strain distribution through thickness, showing the definition of bending and in-plane strain

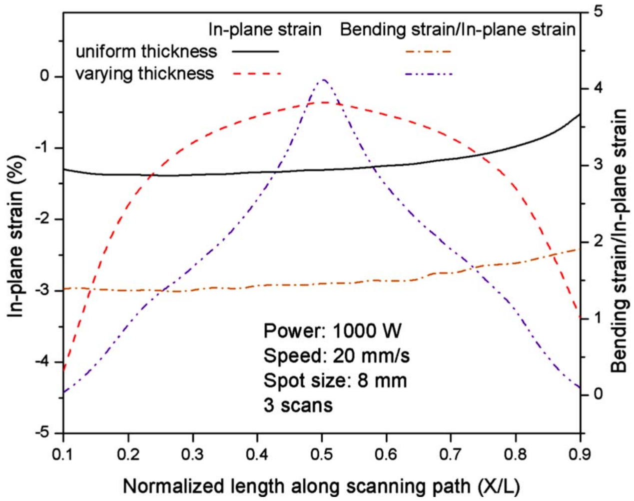

Fig. 7 Distribution of in-plane strain and ratio of bending strain to in-plane strain

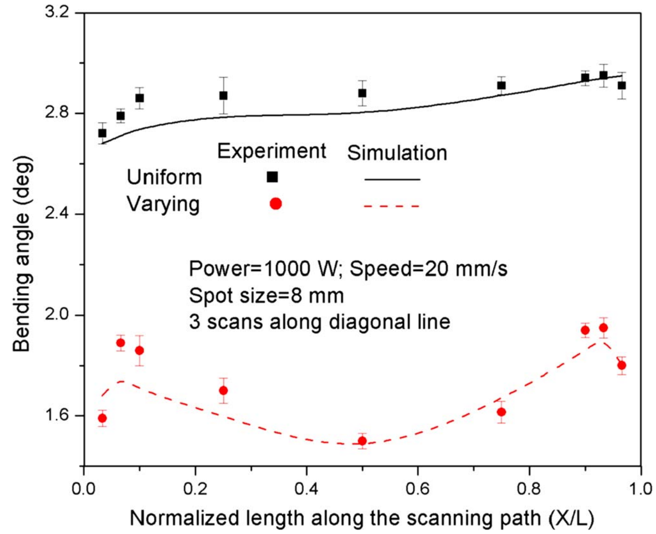

Fig. 9 Bending angle variation along the scanning path when scanned along the diagonal line

the previous section.

Compared with the first case of the scanning varying thickness plate along the centerline, the bending variation is much larger (20%) under the diagonal scanning scheme. From the distribution of peak temperature of the varying thickness plate (shown in Fig. 10), the decrease of peak temperature toward the center of the path is much larger in this diagonal scanning scheme than that of the centerline scanning. The reason is that when scanned along the diagonal line, the heat sink effect increases not only due to the increment of plate thickness, but also due to the increment of dissipated surface area because of the relatively greater distance to the insulative edges. Therefore, the difference of heat sink effect between the ends and the center of the varying thickness plate in diagonal scanning is larger than that of the centerline scanning. For the uniform thickness plate, the increment of heat sink effect, which is only due to the increment of dissipated surface area, makes a much smaller decrement of peak temperature compared with the varying thickness plate.

3.3 Parametric Study. In simulation of the process of laser forming, it is found that certain critical parameters largely determine the temperature distribution and the resulting deformation. In this section, the effects of these parameters on varying thickness plates are studied.

3.3.1 Effect of Beam Spot Size. In this section, the effects of varying spot size on the variation of bending deformation are investigated. Figure 11 shows the bending angle and angle variation under various beam spot sizes. It is seen that the average bending angle decreases when the beam spot size increases. The reason is when the spot size increases, the effective heat input becomes less concentrated, so the temperature inside the heat flux region decreases and the plastic region decreases with a concomitant decrease in bending angle.

It is also found that the variation of the bending angle increases with the increasing spot size. The reason can be found by analyzing the effect of spot size on the temperature distribution, as shown in Fig. 12. From this figure, it is found that both the peak temperature and temperature gradient through the thickness decreases with the increasing beam spot size. The drop of the temperature gradient in the thicker locations is larger than that of the thinner locations. For the thicker locations where the laser forming mechanism is primarily in TGM, the larger drop of temperature gradient causes the bending deformation to drop much more than that of the thinner locations. Since the bending angle in thicker locations is already smaller than that of the thinner locations due to the larger bending rigidity, the variations of bending

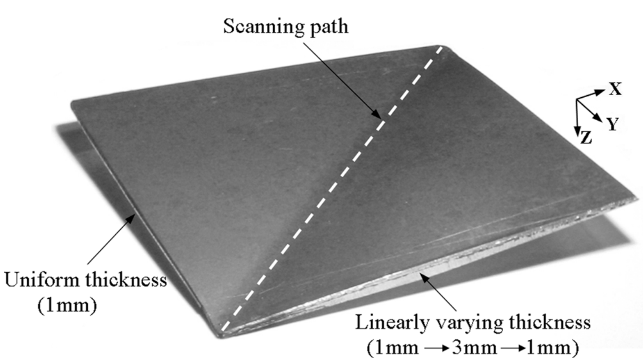

Fig. 8 Experimental result of varying thickness plate scanned along the diagonal line (plate size: 80×80 mm, linearly varying thickness; power=1000 w, scanning speed=20 mm/s, beam spot size=8 mm, number of scans=10)

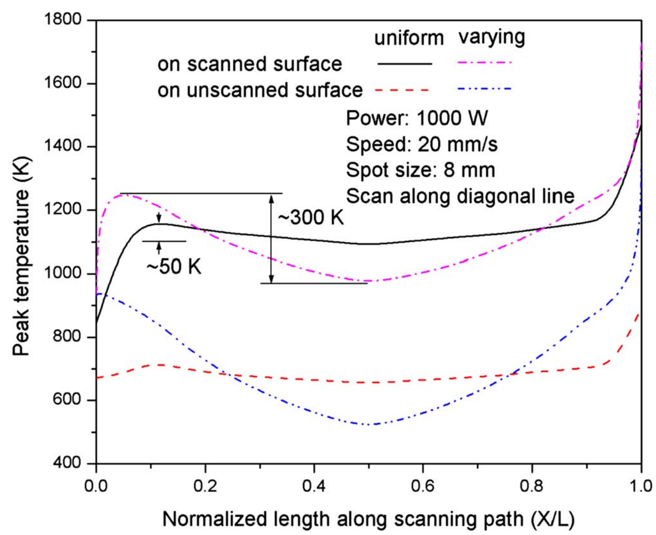

Fig. 10 Comparison of peak temperature along the scanning path between varying and equivalent-uniform thickness plate when scanned along diagonal line

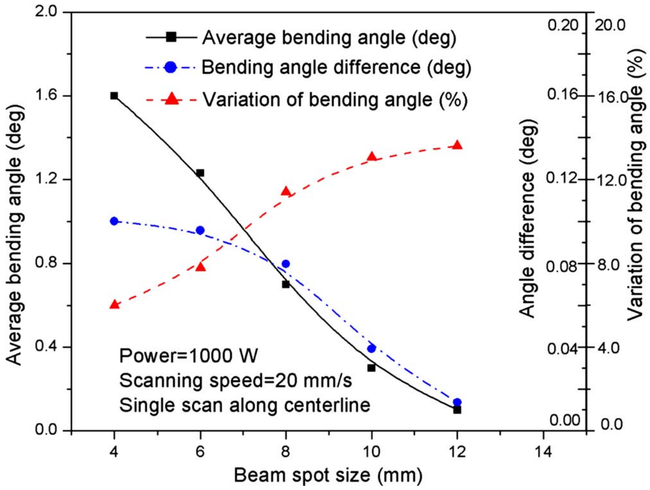

Fig. 11 Bending deformation of varying-thickness coupon under various beam spot sizes (power=1000 W, scanning speed =20 mm/s, single scan along the centerline)

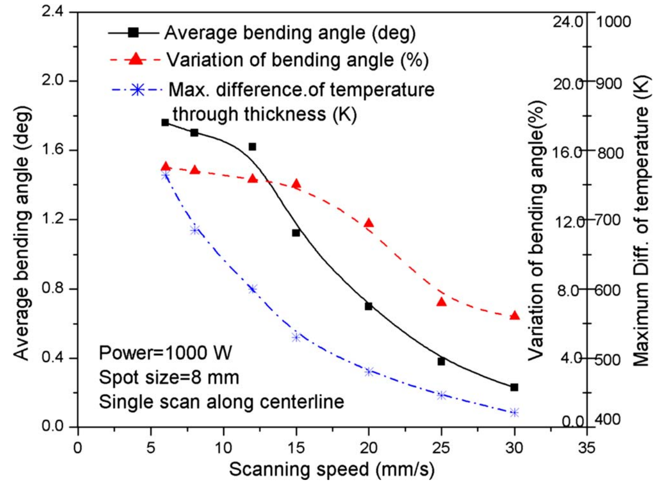

Fig. 13 Bending deformation of varying thickness coupon under various scanning speeds (power=1000 w, spot size =8 mm, single scan along the centerline)

angle will increase with beam spot size. It is also noted that with the beam spot size increasing, the difference of the Fourier number increases, meaning that the laser forming mechanism is more nonuniform along the scanning path. Therefore, the variations of bending deformation in the varying thickness plate will increase with the increasing beam spot size. It can be concluded that smaller beam spot size is helpful to achieve uniform bending deformations on the varying thick plate. But as the beam spot size decreases, higher peak temperature may induce melting on the thinner locations. This restriction can be mitigated by multiple scans.

3.3.2 Effect of Scanning Speed. In this section, the results of a parametric study of the effect of scanning speed on the angular deformation are presented. The heating conditions are similar to the experiments presented in the previous sections, except additional scanning speeds are investigated. Figure 13 shows the bending angle and angle variation under different scanning speed. It shows that generally the average bending angle decreases with increasing scanning speed. The reason is due to the decrease of effective heat input per unit length with the increasing scanning speed, so that both the peak temperature and temperature gradient through the thickness decrease. It also shows that the bending variations decrease with the increasing scanning speeds. The reason is the difference of temperature gradient between thicker locations and thinner locations also decreases with scanning speed (shown in Fig. 13). It is also noted that with the speed increasing, the difference of Fourier number ($F_0=\alpha d/(h^2 \nu)$) decreases and the laser forming mechanism is more uniform to TGM along the scanning path. It also helps to achieve more uniform bending deformations in the varying thickness plate.

3.3.3 Effect of Multiple Scans. To investigate the effect of multiple scans on the bending deformation of the varying thickness plate, multiple scans (from 1 to 10) under the condition of $P=1000$ W, $V=20$ mm/s, $d=8$ mm were carried out. The scans were conducted along the same scanning path with alternate directions in order to reduce the edge effects. Enough time was allowed between scans in both experiments and simulation in order for the material to cool down near the room temperature. Graphite was recoated after every single scan to obtain the same heat input at every scan. Figure 14 shows the average bending angle and angle variation under multiple scans. It is found that the

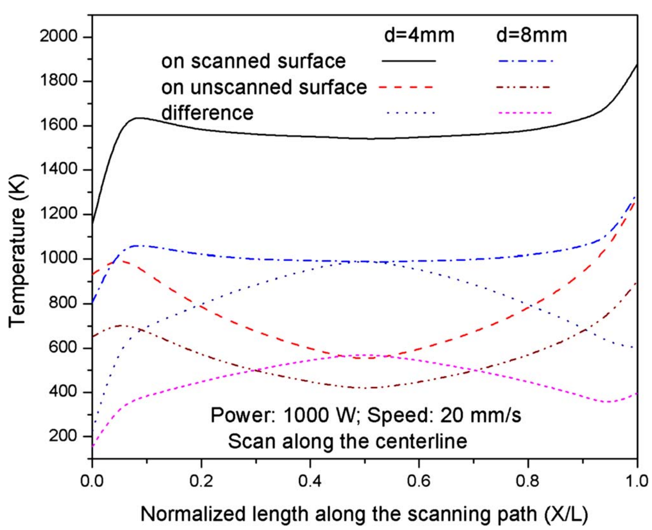

Fig. 12 Temperature distribution of varying thickness coupon under various beam spot sizes (power=1000 W, *d*=4 mm and 8 mm, single scan along the centerline)

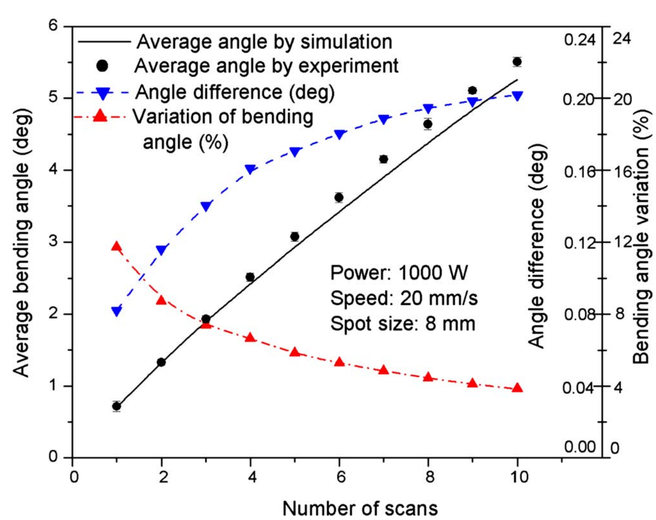

Fig. 14 The relationship between bending angle and number of scans (power=1000 W, scanning speed=20 mm/s, spot size=8 mm)

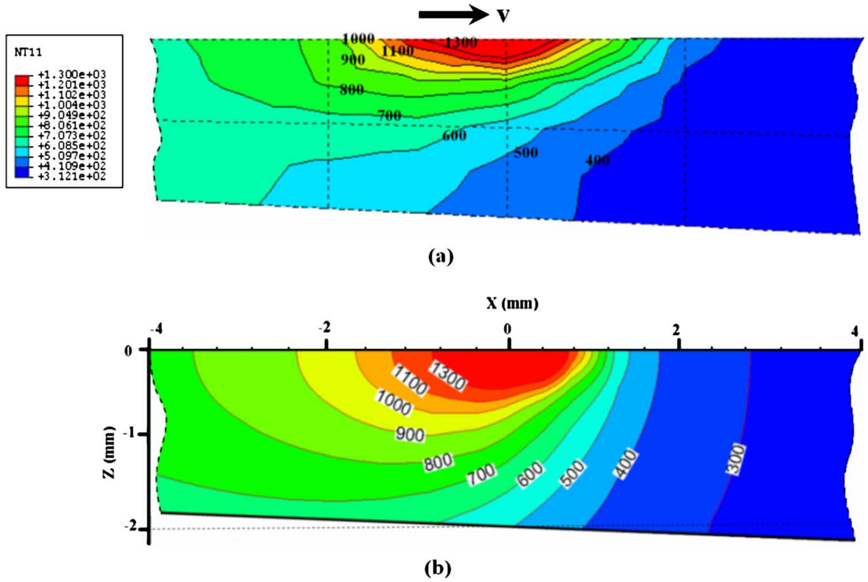

Fig. 15 Temperature field at the cross section along the scanning path of varying thickness plate (*a*) obtained by FEM, and (*b*) obtained by the analytical method (under the condition of power=1000 W, *V*=20 mm/s, the coordinate system refers to Fig. 1)

average bending angle increases with the number of scans and the increment of the bending angle decreases with the number of scans. The reduction in the bending angle increment is due to the work hardening (or strain hardening) and the thickening of the section under irradiation, thereby making the plate increasingly resistant to further deformation.

Under multiple scans, the increase of thickness during processing is not uniform. As discussed in the previous section, the thickness increase is larger at the thinner edges. This non-uniform thickness increase mitigates differences of bending rigidity along the scanning path to decreases the variation of bending deformation. The work hardening is also not uniform in the multiple scans. During each scan, more plastic strain occurs near the thinner edges, as shown in Fig. 4. This (nonuniform) effect of work hardening also helps bring the bending deformation to a more uniform state along the scanning path.

By the above investigation, it can be concluded that for laser forming on the varying thickness plate, the scanning speed, laser beam spot size, and number of scans can be taken as key heating conditions to obtain more uniform bending deformation [13].

4 Further Analysis and Discussion

4.1 Thermal Analysis. In the previous sections, the analysis of laser forming of the varying thickness plate is investigated by experiment and by a fully coupled field-emission microscopy (FEM) simulation. The simulation provides great insight into the transient mechanism of this nonlinear thermomechanical process. However, the computation consumes too much time to be suitable for real-time analysis. A number of simplified thermomechanical models have been derived [5,6,14], which are capable of approximately predicting the amount of bending deformation. To describe the temperature field by laser scanning, an analytical solution to the moving heat source can be applied.

Rosenthal [15] investigated the solutions of the heat equations for a moving plane, moving line, and moving point heat source. When the moving heat source and the heat flux are considered constant, the assumption of a quasistationary heat flow can be made. To apply Rosenthal's solution to the varying thickness plate, the plate can be divided into a number of segments along the scanning directions and the thickness of each segment defined as $h(\mathrm{x})$. Assuming that there is a moving point heat source of q heat units per unit time along the x direction with velocity of V, and there is no loss of heat from the surface of the plate, the corresponding temperature $\theta(x,y,z,t)$ is determined as

$$\theta(x,y,z,t)=\frac{q}{2\pi K h(x)}\left[B_0\left(\frac{Vr}{2\alpha}\right)+2\sum_{n=1}^{\infty}B_0\left(\frac{Vr}{2\alpha}\left(1+\frac{4\pi^2\alpha^2n^2}{v^2h(x)^2}\right)^{1/2}\right)\cos\frac{n\pi z}{h(x)}\right]e^{-V\xi/2\alpha} \tag{1}$$

where $\xi=x-Vt$, $r=\sqrt{\xi^2+y^2+z^2}$, K is the thermal conductivity, $B_0()$ the zero order modified Bessel function, and α the diffusivity. This is the temperature distribution of the infinite plate of finite thickness with a point source moving with constant speed.

The analytical result generally agrees with the FEM simulation result (Fig. 15). Due to the quasistatic assumption, the temperature gradient in the analytical model is a little smaller than the FEM simulation results. Rosenthal [15] proposed that if the solid is long enough as compared to the extent of heat diffusion, the temperature distribution around the heat source soon becomes constant. In other words, the solid has to be long enough to allow the startup transients to die out. However, in reality the transient effect will affect the accuracy coming from this assumption due to the limited length of coupon. In the laser forming process, the rapid travel speed and the relatively "slow" heat conduction speed will cause the temperature gradient to be a little larger than that predicted by analytical model, particularly close to edges.

It is also noted that in the analytical model, the moving heat source is simplified as a moving point source instead of a heat source with a Gaussian distribution, a more accurate description of laser beam shape. The magnitude of the temperature field in the analytical model is a little higher than numerical values due to the neglecting of heat loss at the surfaces through convection. By use of the analytical model, the effect of varying thickness on temperature field can be clearly illustrated: With the thickness increasing, the peak temperature drops down but the temperature difference between the scanned and unscanned surfaces increases. This has been observed and discussed by FEM simulation in previous section.

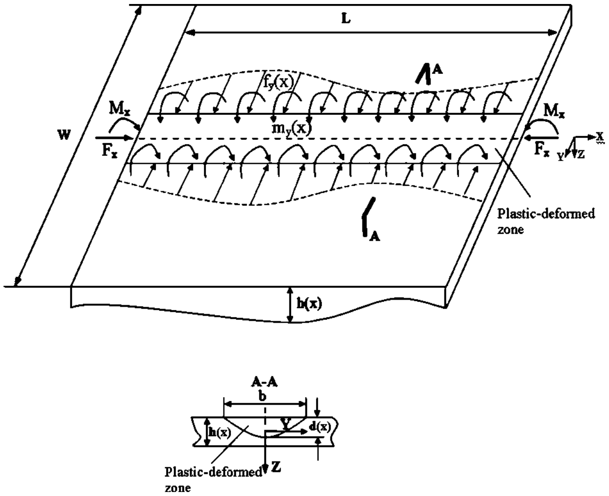

Fig. 16 Schematic of the mechanical model

4.2 Mechanical Analysis. As discussed before, the variation of bending deformation is mainly due to the changing temperature field and the increasing bending rigidity with increasing thickness. In this subsection, a simplified mechanical model is established to describe the bending angle in varying thickness plate.

The equivalent nodal forces and moments of laser forming on a plate with varying thickness along the direction of scanning line are shown in Fig. 16. The transverse bending moment per unit length is expressed as

$$m_y = \int \sigma_y z dz = \int_{-h/2}^{d-h/2} E\epsilon(z) z dz = E\epsilon_{\max}\left(\frac{d(x)h(x)\pi}{8} - \frac{d(x)^2}{3}\right) \tag{2}$$

Transverse shrinkage force per unit length is expressed by

$$f_y = \int_{-h/2}^{d-h/2} E\epsilon(z) dz = \frac{E\epsilon_{\max}}{4} d(x)\pi \tag{3}$$

where d is the depth of heat affected zone (HAZ). $\epsilon_{\max}$ is the maximum plastic strain occurs at the heated surface $\epsilon_{\max} = \alpha_{\rm th}\theta_{\max} - \sigma_y/E$, where $\alpha_{\rm th}$ is the thermal expansion, $\theta_{\max}$ the maximum temperature increment, E the elastic modulus, and σ_y the yield stress.

The bending angle varies along the x direction. It is given by the curvature ρ and breadth of HAZ, b, as $\alpha_B = b/\rho$. From plate theory, the bending curvature at a location with $h(x)$ thickness can be expressed as

$$\frac{1}{\rho} = \frac{m_y}{D} = \frac{12(1-\nu^2)}{Eh(x)^3} m_y \tag{4}$$

where D is the bending rigidity per unit length and defined as

$$D = \frac{E}{1-\nu^2}\int_{h(x)} z^2 dz = \frac{Eh(x)^3}{12(1-\nu^2)} \tag{5}$$

The bending angle of the plate at the location with $h(x)$ thickness is

$$\alpha_B = b(1-\nu^2)\epsilon_{\max}\left(\frac{3d(x)\pi}{2h(x)^2} - \frac{4d(x)^2}{h(x)^3}\right) \tag{6}$$

Determination of the HAZ is important in the calculation of deformation. Jang [16] expressed the size of the heat affected zone based on experimental result as $b = c_1\sqrt{P/V}$ and $d = c_2 P/(Vh)$, where P and V are power and scan speed, respectively. c_1 and c_2 are constants depending on material properties.

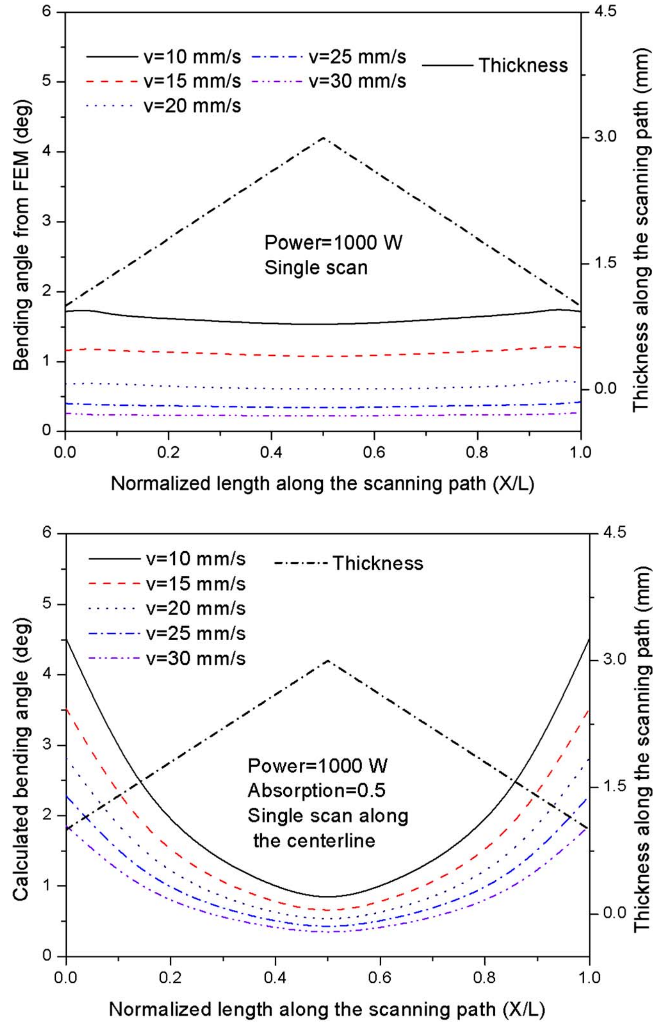

Fig. 17 Bending angle variations with thickness under different scanning speed (*a*) obtained by FEM, and (*b*) obtained by the proposed model

Compared with other theoretical models, the proposed model can predict the bending angle variation in varying thickness plates. It also provides guidance to adjust the heating conditions to achieve more uniform deformation by laser forming. Figure 17 shows the bending angle and angle variation under different scanning speeds obtained by FEM and the proposed model, respectively. It is seen that when the scanning speed increases, the variation of the bending angle decreases. The trend of angle variation with scanning speed agrees well between the proposed model and the numerical results. The angle and angle variation calculated by the proposed model are larger than that by FEM although they are in the same order. The former discrepancy is due to the neglecting of the heat loss and the over-estimated dimensions of the plastic-deformed zone in the proposed model. The latter discrepancy is also due to neglecting of the counteraction between adjacent locations along the scanning direction in the proposed model.

5 Conclusions

For laser forming of varying thickness plates, the bending variation along the scanning path may affect the quality of production and cannot be ignored. The variation of the bending angle is primarily due to the variation of the heat sink and the bending rigidity. With the thickness increasing, temperature gradient mechanism (TGM) dominates and the bending strain plays a more important role. Scanning speed and beam spot size are key heating parameters to reduce the bending variation because the laser forming mechanism is primarily dependent on them except for plate thickness. Smaller beam spot size may decrease bending variation because of the larger increments of temperature gradient through the thickness direction in the thicker locations of the plate. Higher scanning speed is also helpful to achieve more uniform bending deformations for the varying thickness plate, because the difference of peak temperature and temperature gradient between thicker and thinner locations will decrease with increasing scanning speed. Multiple scans under given heating conditions can also be used to achieve more uniform bending deformation. In the further analysis, Rosenthal's solution for a moving point heat source on an infinite plate was applied in the thermal model and the variable bending rigidity was considered in the mechanical model. The proposed model can capture the trend of bending angle and angle variation well. Therefore, it provides another tool to analyze and predict trends in the bending angle and its variations due to varying thickness.

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